

Far Western University Faculty of Education B.Ed. in Mathematics Education

#### Course Title: Differential Calculus

Course No. : Math.Ed.111 Semester: First Credit Hour: 3 (45 hours) Level: B. Ed. Full marks: 100 Pass marks: 45

#### **1.** Course Introduction

This course is designed for undergraduate students to develop acquaintance with fundamental principles, approaches and techniques of differential calculus. It helps students to create a foundation of higher mathematical courses such as Analysis. Starting with the basic concepts of function, limits, continuity and derivatives, the course covers Mean Value Theorems, partial differentiations, tangents and normal, extreme values, curvature, asymptotes, and curve tracing. Whilst the due emphasis is given to conceptual understanding and problem investigation, students will experience some key application areas in the learning process of this course.

#### 2. General Objectives

General objectives of this course are as follows:

- To demonstrate understandings and skills of various concepts, principles and approaches of differential calculus.
- To apply concepts and skills of differential calculus in solving problems of different branches of mathematics.
- To use mean value theorems in writing functions in expanded form.
- To find limits concerning indeterminate form.
- To demonstrate understanding and skills on extreme values, tangent and normal, curvature, partial derivatives and asymptotes.
- To sketch curves of different types of functions using the concepts of differential calculus.
- To appreciate the role of differential calculus in solving problems of different disciplines.
- To become confident on their learning of various concepts, principles and approaches of differential calculus.

### 3. Contents in Detail with Specific Objectives

Specific Objectives	Contents
• To explain the concept of a function.	Unit 1: Function, Limit and Continuity (2 hours)
• To discuss the idea of domain and graph of a	
function.	1.1 Concept of a function
<ul> <li>To explore the relationship between ε-δ definition of a limit of a function and its</li> </ul>	1.2 Definition of a limit of a function
geometrical interpretation.	1.3 Continuity and discontinuity of a function
• To explore the relationship between continuity of a function at a point and its geometrical	1.4 Geometrical meaning of limit and continuity of a
interpretation.	function
• To evaluate limits of functions and determine	
whether a function is continuous at a point and	
on an interval.	

• To define derivative of a function at a	Unit 2: Derivatives (4 hours)	
point and interpret it geometrically.	Chit 2. Derivatives (4 hours)	
• To find derivative of different types of	2.1 Concept of a derivative of a function	
functions.	2.2 Derivatives of different type of functions	
• To explain the concept of higher order derivatives.	2.3. Concept of higher order derivatives	
• To find the higher order derivatives of	2.4. n <sup>th</sup> order derivatives of the functions such	
<ul><li>some functions.</li><li>To state and prove the Leibnitz theorem.</li></ul>	as: $x^{n}$ , $(ax + b)^{n}$ , $sin(ax + b)$ , $log(ax + b)$ etc.	
<ul> <li>To solve the problems using Leibnitz</li> </ul>	2.5 Leibnitz theorem and its application	
theorem.	2.5 Letomiz deorem and its application	
• To interpret meaning of Roll's theorem, Lagrange's theorem, Cauchy's theorem; Taylor's theorem, and Maclaurin's series	Unit 3: Mean Value Theorem and its applications (7 hours)	
and prove the theorems.	3.1 Roll's Theorem	
• To interpret geometrically the meaning of Rolle's theorem, Lagrange's theorem, and	3.2 Lagrange's mean value theorem	
Cauchy's theorem.	3.3 Cauchy's mean value theorem	
• To verify Rolle's theorem, Lagrange's	3.4 Taylor's theorem with Lagrange and Cauchy	
theorem, and Cauchy's theorem for some functions.	form of remainders (finite and infinite form)	
<ul> <li>To expand some functions by using Maclaurin's series.</li> </ul>	3.5 Maclaurin's series	
<ul> <li>To be confident on their learning of above mentioned theorems.</li> </ul>		
• To state and explain different types of indeterminate forms.	Unit 4: Indeterminate forms ( 3 hours)	
• To state, prove and generalize the L hospital's theorem.	<ul><li>4.1 Different indeterminate forms</li><li>4.2 L'hospital's theorem</li></ul>	
• To calculate the limits of functions of various indeterminate forms.	4.3 Limits of functions of indeterminate forms	
• To appreciate the role of L'hospital's theorem on calculating limits.		
• To calculate limits and check continuity	Unit 5: Partial Differentiation ( 6 hours)	
of functions of two variables.	5.1 Limits and continuity of functions of two	
• To define partial derivatives and interpret geometrically the partial derivatives of	variables	
first order of two variables.		
• To explore the relationship between definition and geometrical representation.	5.2 Definition of partial derivatives	
<ul> <li>To calculate partial derivatives of higher order.</li> </ul>	5.3 Geometrical interpretation of partial derivatives of first order	
<ul><li>To state, verify and use the Euler's</li></ul>	5.4 Partial derivatives of higher order	
theorem on homogeneous functions.	5.5 Euler's theorem on homogeneous functions on	
• To find the derivatives of composite functions.	two variables	
<ul> <li>To find the derivatives of implicit</li> </ul>		
functions using partial derivatives.	5.6 Derivatives of composite functions	
<ul> <li>To become engaged on the learning of different concepts and skills of partial derivatives.</li> </ul>	5.7 Derivatives of implicit functions	

<ul> <li>To derive equation of tangents and normal of curves in different forms (explicit, implicit and parametric forms).</li> <li>To find the angle of intersection of two curves in Cartesian/polar forms.</li> <li>To find the length of sub/tangent, sun/normal in Cartesian and polar forms</li> <li>To calculate the derivatives of arc length in Cartesian and polar forms.</li> <li>To derive Pedal equation of Cartesian and polar curves.</li> <li>To show confidence on the learning of different concepts and skills associated with tangent and normal.</li> </ul>	<ul> <li>Unit 6: Tangent and Normal ( 4 hours)</li> <li>6.1 Equation of tangent and normal</li> <li>6.2 Angle of intersection of two curves (Cartesian and polar forms)</li> <li>6.3 Length of sub/tangent, sub/normal (Cartesian and polar forms)</li> <li>6.4 Derivatives of arc length (Cartesian and polar forms)</li> <li>6.5 Pedal equation of Cartesian and polar curves</li> </ul>
<ul> <li>To define and identify the increasing and decreasing functions, concavity and convexity, stationary points, point of inflections and saddle points.</li> <li>To appreciate the role of derivative from the applied aspects.</li> <li>To prove the necessary and sufficient conditions for maximum and minimum of the functions.</li> <li>To explain the various conditions for extreme values while solving problems.</li> <li>To use Lagrange's methods of undetermined multipliers whilst calculating maximum/minimum values.</li> <li>To solve various problems related to maxima and minima.</li> </ul>	<ul> <li>Unit 7: Maxima and Minima ( 6 hours)</li> <li>7.1 Increasing and decreasing functions, concavity and convexity, stationary points, point of inflections and saddle points</li> <li>7.2 Conditions for maximum and minimum of functions (up to three variables)</li> <li>7.3 Extreme values under various constraints</li> <li>7.4 Lagrange's methods of undetermined multipliers</li> </ul>
<ul> <li>To explain the meaning of curvature and radius of curvature.</li> <li>To derive formula for radius of curvature in different forms and apply them to solve related problems.</li> <li>To find radius of curvature at origin by using different methods.</li> <li>To derive expression for length chord of curvature (through pole &amp; parallel to solve coordinate axes) &amp; apply them to solve related problems.</li> <li>To explain circle of curvature</li> <li>To derive expression for center of curvature and apply it in solving related problems.</li> <li>To show confidence on the learning of different concepts and skills associated with curvature.</li> </ul>	<ul> <li>Unit 8: Curvature (6)</li> <li>8.1 Concept of curvature and radius of curvature</li> <li>8.2 Formula for radius of curvature in different forms: (Intrinsic form, Cartesian form, Parametric form, Polar form, Pedal form and Tangential polar form)</li> <li>8.3 Curvature at origin</li> <li>8.4 Chord of curvature(Through origin, Parallel to coordinate axes )</li> <li>8.5 Circle of curvature</li> <li>8.6 Centre of curvature</li> </ul>

<ul> <li>To explain a meaning asymptotes and represent them in a graph.</li> <li>To determine horizontal, vertical and oblique asymptotes.</li> <li>To find the asymptotes of some algebraic and polar curves</li> </ul>	<ul> <li>Unit 9: Asymptotes (3 hours)</li> <li>9.1 Definition of asymptotes, its representation in graph</li> <li>9.2 Horizontal, vertical and oblique asymptotes</li> <li>9.3 Asymptotes of algebraic and polar curves</li> </ul>
<ul> <li>To illustrate the properties of the curve while sketching it.</li> <li>To sketch the curves of functions in Cartesian and Polar forms.</li> </ul>	<ul> <li>Unit 10: Curve Sketching ( 4 hours)</li> <li>10.1 Properties for curve Sketching (symmetry, origin, noticeable points, tangents at origin,</li> </ul>
<ul> <li>To appreciate the role of differential calculus in curve tracing.</li> </ul>	points of inflections, concavity and convexity, asymptotes) 10.2 Curve Sketching of some functions

### 4. Methodology and Techniques

- Inquiry Based Learning to derive formulae and to develop conceptual understanding.
- Project-Based Learning to facilitate application aspect.
- Problem Based Learning to help students in solving problems in the exercises.
- Support students in their ZPD using constructivist perspective.
- Exploration: Help students to explore the essence of the contents, prove the necessary theorem, and solve problems.
- Use collaborative learning methods together with expository-based demonstration methods as per the nature of the content.
- Discussion: discuss the application of the theorems and ask students to solve the problems.
- As far as possible teacher need to focus on authentic and meaningful learning by taking help of reference books.
- Teachers may use mathematical software (e.g., MATLAB)

## 5. Evaluation Scheme

## 5.1 Internal Evaluation (40%)

Internal Evaluation will be conducted by course teacher based on following activities.

a) Attendance and Participation in class activities:	5+5= 10 marks
b) Assignment I: Reflective Notes and Class presentation: (Reflective notes on 2 to 4 questions given by teacher at the end of the every unit and presentation on any two questions among them )	5+5= 10 marks
c) Assignment II: one Term paper/ Essay/Project and Interview: (Logical essay/term paper/project on the topics chosen by students and approved by the teacher and interview)	5+5=10 marks
d) Mid-term exam:	10 marks

### **Description of the Internal Evaluation**

*Mid-term exam: Engagement in a Class:* Marks will be assigned based on the attendance and engagement in the classroom activities. At least 80% percent class attendance is mandatory for the students to enable them to appear in the End-Term examination. Below 80% in attendances that signify is NOT QUALIFIED (NQ) in subject to attend the end term examination.

*Reflective Journal:* It is individual work. Each student must submit their reflective journal of each chapter or teacher will give some questions that need reflective activities. The reflective journal will be

returned to the students after its evaluation. Each student need to make presentation on their reflective journal.

*Term paper:* It is individual work. It must be prepared by the use of computer in a standard format of academic writing and must contain at least 5 pages. Quality, format, and time of submission will be the major criteria of the evaluation. Teacher will take interview of students based on their term paper.

**Project Work:** Students will be divided into groups. Each group will be assigned the project concerning application of theorems. Each group will present their findings in a whole class.

*Mid-Term Examinations:* It is a written examination and the questions will be set covering the topics as taught in the sessions. Mid-term examination will be based on the model prescribed for End-term examination.

## 5.2 External Evaluation (60%)

**External Examinations:** It is also a written examination and the questions will be asked covering all the topics in the session of the course. It carries 60 marks.

Nature of question	Total questions to be asked	Total questions to be answered	Total marks
Group A: Multiple choice	10 questions	10	$10 \times 1 = 10$
<b>Group B:</b> Short answer type question	6 with 2 'or' questions	6	6×5 = 30
<b>Group C:</b> Long answer type question/case studies	- Vivith L'or' dilection		2×10=20
Total		60	

### **End Semester Examination Model**

## **Recommended book**

Koirala, S. P., Pandey, U. N., Pahari, N. and Pokhrel, P. (2010). *A textbook on differential calculus*. Vidyarthi Prakashan.

## References

Das, B. C. & Mukherjee, B. N. (1994). Differential Calculus (40th ed.). Narosa Publishing House

Larson, R., & Edwards, B. H. (2009). Calculus (9th ed.). Brooks/Cole.

Spivak, M. (2008). Calculus. New York: Cambridge University Press.

Thomas, G.B. & Finney, R.L. (2001). Calculus (9th edition). Pearson Education.



# Far Western University Faculty of Education B.Ed. in Mathematics Education

## Course Title: Probability and Statistics

Course No. : Math. Ed. 112 Level: B. Ed. Semester: First Nature of the course: Theory Total periods: 45 Time per period: 1 Hour

# 1. Course Introduction

The main aim of this course is to make students familiar with concepts, skills, and applications of probability theory and inferential statistics. The probability theory is foundation for the inferential statistics and the inferential statistics deals with estimation and hypothesis testing. The contents in this course include various probability distributions, estimations, correlation and regression, and hypothesis testing.

# 2. General Objectives

The general objectives of this course are as follows:

- To demonstrate a conceptual understanding of probability and probability distributions.
- To demonstrate understanding and skills of sampling and estimation
- To apply the concept of correlation and regression in solving problems.
- To demonstrate understanding of hypothesis testing approaches.
- To apply z-test, t-test, and chi-square test in solving problems of daily life concerning hypothesis testing.
- To appreciate the role of inferential statistics in hypothesis testing.
- To be engaged in applying concepts of probability and inferential statistics in solving problems related to various areas.
- To be confident on the learning of skills, concepts, formulae and applications of probability and inferential statistics

# 2. Contents in Detail with Specific Objectives

Specific Objectives	Contents
<ul> <li>To explain meaning of different terminologies concerned with probability.</li> <li>To compare meaning of a probability through mathematical, empirical and axiomatic approach.</li> <li>To explain and prove addition, multiplication and Bayes's theorem.</li> <li>To solve fundamental problems using above theorems.</li> <li>To show confident the learning of concepts and skills of fundamentals of probability.</li> </ul>	<ul> <li>Unit I: Fundamentals of Probability (4 hrs)</li> <li>1.1 Terminologies of probability</li> <li>1.2 Concept of a probability (Mathematical, Empirical, and axiomatic approach)</li> <li>1.3 Axiomatic probability</li> <li>1.4 Theorems of probability (Addition theorem, Multiplication theorem, Bayes's theorem)</li> </ul>
<ul> <li>To describe the concept of discrete random variable and its probability distribution, mathematical expectation, mean and variance.</li> <li>To explain the concept of binomial distribution and its properties.</li> </ul>	<ul> <li>Unit II Discrete Probability Distributions (5 hrs)</li> <li>2.1 Discrete Random Variable: Probability distribution, mathematical expectation, mean and variance</li> <li>2.2 Binomial distribution: Concept, definition,</li> </ul>

<ul> <li>To explain the concept of binomial distribution and its properties.</li> <li>To solve problems associated with binomial and Poisson distribution.</li> <li>To compare the binomial distribution and Poisson distribution.</li> </ul>	properties, related problems 2.3 Poisson distribution: Concept, definition, properties, related problems
<ul> <li>To describe continuous random variable and its probability density function.</li> <li>To explain concept, probability density function, and properties of normal distribution.</li> <li>To explain the concept of a standard normal distribution in relation to normal distribution.</li> <li>To solve problems concerning area under a standard normal curve.</li> <li>To explore the relation between Binomial and Normal distribution.</li> <li>To appreciate the role of standard normal distribution in solving daily life problems.</li> </ul>	<ul> <li>Unit III: Normal Distribution (5 Hrs)</li> <li>3.1 Continuous Random Variable and its probability density</li> <li>3.2 Concept, probability density function, and properties of a Normal distribution:</li> <li>3.3 Concept of a standard normal distribution</li> <li>3.4 Areas under standard normal curve</li> <li>3.5 Relationship of Normal distribution with binomial distribution and Poisson distribution</li> </ul>
<ul> <li>To explain the concept of a population and sample.</li> <li>To describe methods of sampling</li> <li>To describe the concept of parameter and statistics.</li> <li>To discuss the sampling distribution and standard error of statistics.</li> <li>To explain the sampling distribution of mean.</li> <li>To differentiate sampling and non-sampling errors.</li> <li>To describe the meaning and application of the central limit theorem.</li> <li>To appreciate the role of central limit theorem in statistics.</li> </ul>	<ul> <li>Unit IV Sampling Theory (4 hrs)</li> <li>4.1 Population and sample</li> <li>4.2 Sampling Methods (simple random, stratified random, systematic, cluster)</li> <li>4.2 Parameters and statistics</li> <li>4.3 Concept of a sampling distribution of a statistic</li> <li>4.5 Sampling distribution of mean</li> <li>4.6 Standard Error of some statistics</li> <li>4.7 Sampling Error and Non-sampling error</li> <li>4.7 Central Limit Theorem</li> </ul>
<ul> <li>To explain the concept of point estimation and interval estimation.</li> <li>To compare the properties of a good estimator (unbiasedness, consistency, efficiency, sufficiency).</li> <li>To determine confidence interval for mean, proportion, variance, difference of means, and difference of proportions.</li> <li>To be engaged in finding interval estimate.</li> </ul>	<ul> <li>Unit V: Theory of Estimation (5 hrs)</li> <li>5.1 Concept of a Point Estimation</li> <li>5.2 Criteria of a good estimator</li> <li>5.3 Meaning of interval estimation</li> <li>5.4 Confidence intervals for mean, proportion, and variance</li> </ul>
<ul> <li>To explain the concept of correlation and regression.</li> <li>To find Pearson's correlation, rank correlation, and regressions.</li> <li>To apply correlation and regression in solving problems.</li> </ul>	<ul> <li>Unit VI Correlation and Regression (4 hrs)</li> <li>6.1 Properties of correlation, probable error</li> <li>6.2 Pearson's Correlation</li> <li>6.3 Rank Correlation</li> <li>6.4 Equation of Regression, properties of regression</li> </ul>

<ul> <li>To explain the concept of some basic terminologies of Hypothesis testing.</li> <li>To describe steps of hypothesis testing using p-value approach and critical value approach.</li> <li>To explore the relationship between two approaches of hypothesis testing.</li> </ul>	<ul> <li>Unit VII: Introduction of Hypothesis Testing <ul> <li>(4 hrs)</li> </ul> </li> <li>7.1 Basic Concept: Meaning and Characteristics of <ul> <li>Hypothesis, Null and Alternate hypothesis,</li> <li>One-tailed and two-tailed test, Type I and Type <ul> <li>II error, Level of significance and critical region,</li> <li>parametric and non-parametric test</li> </ul> </li> <li>7.2 Steps in Test of Hypothesis: p-value approach <ul> <li>and critical value approach</li> </ul> </li> </ul></li></ul>
<ul> <li>To compare the conditions for z test, t-test, chi-square test, and f-test.</li> <li>To test hypothesis concerning single mean, single proportion, difference between means, difference between proportion using z-test or t-test.</li> <li>To test hypothesis concerning single variance, goodness of fit, and independence of attributes using Chi-square test.</li> <li>To test hypothesis concerning equality of variances using f-test.</li> <li>To appreciate the role of different tests in solving problems concerning daily life.</li> <li>To be engaged in the projects concerning decision making by testing hypothesis.</li> </ul>	<ul> <li>Unit VIII: Testing of Hypothesis (14 Hrs.)</li> <li>8.1 Assumptions and applications of z-test, t-test, Chi-square test, f-test</li> <li>8.2 Test of significance for single mean, difference between two means (independent samples and correlated samples), single proportion, and difference between two proportions.</li> <li>8.3 Test of significance concerning single variance, goodness of fit and independence of attributes</li> <li>8.4 Test of significance of equality of variances</li> </ul>
<ul> <li>Use collaborative learning methods together we the nature of the content.</li> <li>Discussion: discuss the application of the theory applying theorems.</li> </ul>	on aspect. olving problems in the exercises.

• Teachers may use mathematical software SPSS.

# 5. Evaluation Scheme

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Group B: Short answer type question	6 with 2 'or' questions	6	6×5 = 30
Group C: Long answer type question	2 with 1 'or' question	2	2×10=20
Total			60

#### **End Semester Examination Model**

#### **Recommended/Prescribed Book**

Sthapit, A., Yadav, R., Khanal, S., & Dangol, P.(2014). Applied Statistics. Asmita Publication:

#### References

David, S. (1999). Probability and random variables: A beginner's guide. Cambridge University Press.

Freund, J. E. (2009). Mathematical statistics with application. Pearson Education.

Gupta, S. C. (2006). Fundamental of statistics. Himalaya Publishing House.

Gupta, S. P. (2007). Statistical Method. S. Chand and Sons Publishers.