Far Western University
Faculty of Education
B. Ed. in Mathematics Education

Course Title: Number Theory
Course No: Math.Ed.233
Level: Undergraduate
Semester: Third

Nature: Theoretical
Credits: 3
Teaching hours: 48

1. Course Description

This course is designed for undergraduate-level students to develop understanding and skills associated with number theory. The whole course deals with the theory of integers. The course is important for undergraduate students because integers play a very important role in any field of mathematics and number theory provides a basis for algebra, analysis, and other courses. The course deals with concept, skills, principles, and constructing proof of theorems concerning integers, divisibility in the integers, the fundamental theorem of arithmetic, congruence, number theoretic functions, and quadratic reciprocity law. Simple arithmetical skills, logical skills, and knowledge of proof techniques are prerequisites of the course.

2. General Objectives

The main objective of the course is to make students able to understand and apply different concepts, skills, and principles of number theory. The general objectives of this course are as follows:

- To develop an understanding of a set of integers and its basic properties.
- To make students able to apply the divisibility theory of integers in solving problems related to numbers.
- To make students able to apply fundamental theorem of arithmetic in proving theorems and solving problems.
- To develop an understanding of number theoretic functions and be able to prove related theorems.
- To develop an understanding of different types of numbers and apply them in solving problems.
- To develop an understanding of congruence and apply related theorems to solve problems from number theory.
- To develop attitudes of appreciating the beauty of the theory of integers.
- To engage students in developing proofs and applying theorems.

3. Objectives and contents

Specific Objectives	Contents
• To explain the concept of integers, rational numbers, irrational numbers, algebraic numbers and transcendental numbers with examples. • To state the well-ordering principle with examples. • To differentiate between countable and uncountable sets. • To explain sequences particularly geometric and	**Unit I: The Integers (4)** 1.1 Numbers 1.2 Well ordering principle 1.3 Countable sets and uncountable sets 1.4 Sequences 1.5 Sums and products 1.6 Mathematical induction

arithmetic progressions. • To use summation and product signs appropriately and evaluate sum and products. • To state and prove first and second principles of mathematical induction and apply them. • To explain the nature of recursive definitions. • To evaluate the value of greatest integer functions.	1.7 Recursive definitions 1.8 Greatest integer function
• To describe the concept of divisibility in the integers with examples. • To state and prove division algorithm and apply it to solve problems. • To define *gcd* and *lcm* of integers; prove their properties; and apply properties to solve problems. • To state and prove Euclidean algorithm with example. • To derive expression for solution of linear Diophantine equation and apply it in solving equations.	**Unit II: Divisibility Theory in the Integers (7)** 2.1 Concept of divisibility 2.2 The division algorithm 2.3 The greatest common divisor 2.4The Euclidean algorithm 2.5 The least common multiple 2.6 The linear Diophantine equation
• To describe the concept of prime and composite numbers with examples. • To prove some theorems concerning division by prime numbers. • To state and prove fundamental theorem of arithmetic; apply it to solve problems concerning gcd and lcm. • To explain and use Sieve of Eratosthenes.	**Unit III: Primes and their distribution(4)** 3.1 Concept of prime and composite numbers 3.2 Fundamental theorem of Arithmetic 3.3 The sieve of Eratosthenes
• To define congruence with example. • To prove basic properties of congruence and apply them to solve problems. • To prove property of linear congruence & apply in problems. • To state, prove, and apply Chinese remainder theorem. • To apply concept of congruence in testing divisibility of integers. • To state and prove Fermat's theorem • To state and prove Wilson's theorem • To apply these theorems to solve problems • To develop a project to determine day of any year using congruent modulo 7.	**Unit IV: The Theory of Congruence (9)** 4.1 Definition of congruence 4.2 Basic properties of Congruence 4.3 Complete set of residues 4.4 Linear congruence and Chinese remainder theorem 4.5 Divisibility tests 4.6. Fermat's little theorem 4.7 Wilson's theorem 4.8 Application in Calendar
• To explain the concept of number theoretic functions τ, σ, μ , and $\emptyset$. with example. • To prove properties of number theoretic functions τ, σ, μ , and $\emptyset$. • To prove that functions τ, σ, μ , and $\emptyset$ are multiplicative.	**Unit V: Numbers theoretic functions (11)** 5.1 The functions τ and σ 5.2 Basic properties of τ and σ 5.3 The Mobius μ function 5.4 Euler's phi function 5.5Basic properties of $\emptyset$ function

• To state and prove Euler's generalization of Fermat's theorem. • To evaluate the value of number theoretic functions using properties.	5.6 Multiplicative nature of τ, σ, and Ø functions 5.7 generalized form of Fermat's theorem(Euler's theorem)
• To explain the concepts of order of an integer and primitive roots of an integer with examples. • To define quadratic residues & non- residues with example. • To prove some basic theorems associated with quadratic residues. • To state and prove Euler's criterion about quadratic residues • To define Legendre symbol with example and prove their properties. • To state and prove Gauss lemma and related theorem. • To state and prove quadratic reciprocity law. • To apply quadratic reciprocity law in solving associated problems.	**Unit VI: Quadratic Reciprocity law** (8) 6.1 Primitive roots of an integer 6.2 Quadratic residues and non-residues 6.3 Euler's criterion 6.4 The Legendre symbol and their properties 6.5 Gauss lemma and related theorems 6.6 Quadratic reciprocity law
• To describe concept of perfect numbers , format numbers, Mersene primes, and Fibonacci numbers with examples. • To state and apply some basic properties (without proofs) of perfect numbers , format numbers, and Mersene primes, • To prove some properties of Fibonacci numbers.	**Unit VII: Numbers of special forms (5)** 7.1 Perfect numbers and their properties 7.2 Fermat's numbers and their properties 7.3 Mersene primes and their properties 7.4 Fibonacci numbers and their properties

4. Methodology and Techniques

- To facilitate the concepts of the number theory better to start from examples.
- Demonstrate the meaning of theorem with help of example and then apply Inquiry Based Learning to develop proof of the theorems.
- Teachers are suggested to engage students in illustrating theorems by taking suitable examples.
- **Constructivist approach** to develop conceptual understanding of concepts.
- **Project-Based Learning** to facilitate application aspect of theorems.
- **Problem Based Learning** to help students in solving problems in the exercises.
- Support students in their **ZPD** using constructivist perspective.
- **Exploration**: Help students to explore the essence of the concepts and theorems.
- Emphasize collaborative learning methods.
- Expository-based demonstration methods might be helpful in some content.

- **Discussion:** discuss the application of the theorems/formulas and ask students to solve the problems applying theorems.
- **Use of references:** Rosen, K. H. [for Unit I, 4.5, 4.8. as main text and for other topics as supportive text] and Burton, D. M. [for all units except unit I as main text]. Solved examples and exercise problems can be selected from any of the text books and proof of theorems can be selected from any of the books.

5. Evaluation Scheme

5.1 Internal Evaluation (40%)

Internal Evaluation will be conducted by course teacher based on following activities.

a) **Attendance and Participation in class activities**: **5+5= 10 marks**

b) **Assignment I: Reflective Notes and Class presentation:** **5+5= 10 marks**
(Reflective notes on 2 to 4 questions given by teacher at the end of the every unit and presentation on any two questions among them)

c) **Assignment II**: **one Term paper/ Essay/Project and Interview:** **5+5=10 marks**
(Logical essay/term paper/project on the topics chosen by students and approved by the teacher and interview)

d) **Mid-term exam:** **10 marks**

Description of the Internal Evaluation

Mid-term exam*:* ***Engagement in a Class:*** Marks will be assigned based on the attendance and engagement in the classroom activities. At least 80% percent class attendance is mandatory for the students to enable them to appear in the End-Term examination. Below 80% in attendances that signify is NOT QUALIFIED (NQ) in subject to attend the end term examination.

Reflective Journal: It is individual work. Each student must submit their reflective journal of each chapter or teacher will give some questions that need reflective activities. The reflective journal will be returned to the students after its evaluation. Each student need to make presentation on their reflective journal.

Term paper*:* It is individual work. It must be prepared by the use of computer in a standard format of academic writing and must contain at least 5 pages. Quality, format, and time of submission will be the major criteria of the evaluation. Teacher will take interview of students based on their term paper.

Project Work: Students will be divided into groups. Each group will be assigned the project concerning application of theorems. Each group will present their findings in a whole class.

Mid-Term Examinations: It is a written examination and the questions will be set covering the topics as taught in the sessions. Mid-term examination will be based on the model prescribed for End-term examination.

5.2 External Evaluation (60%)

External Examinations: It is also a written examination and the questions will be asked covering all the topics in the session of the course. It carries 60 marks.

End Semester Examination Model

Nature of question	Total questions to be asked	Total questions to be answered	Total marks
Group A: Multiple choice	10 questions	10	10×1 = 10
Group B: Short answer type question	6 with 2 'or' questions	6	6×5 = 30
Group C: Long answer type question/case studies	2 with 1 'or' question	2	2×10 =20
Total			**60**

References

Burton, D. M. (2012). *Elementary Number Theory (7th ed.).* New Delhi: Tata McGraw Hill Education Private Limited

Rosen, K. H. (2005). *Elementary Number Theory and its Applications (5th).* London: Pearson.

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B. Ed. in Mathematics Education

Course Title: Basic Linear Algebra
Course No: Math.Ed.234
Level: Undergraduate
Semester: Third

Nature: Theoretical
Credits: 3
Teaching hours: 48

1. Course Introduction

The course *Basic Linear Algebra* deals with matrices, systems of linear equations, vector spaces, and maps between vector spaces. The concepts and skills in the course are helpful for undergraduate students of mathematics education. This is because the content of the course is useful for teaching algebra at the secondary level. It is equally important for students to study in the field of linear algebra. The course focuses on concepts, skills, principles, theorem proving, and applications. There are nine chapters in the course starting from linear equations. The course begins with the concept of matrices and then the system of linear equations is introduced relating with matrices. Then the liner geometry will be introduced to help students form visual imagery and the foundation of more abstract vectors in spaces. With the help of linear maps and isomorphism, the vector spaces will be studied. The course ends with the concept and skills of eigenvalues and eigenvectors. The prerequisites of the courses are simple arithmetical and algebraic skills, concept of functions, concept of real numbers, logical ideas, and knowledge of proof techniques.

2. Course Objectives

The main aim of the course is to develop concepts and skills in matrices, systems of linear equations, and vector spaces. The general objectives of the course are as follows:

- To develop understanding and skills in matrix operations and determinants.
- To develop skills in solving systems of linear equations with Gaussian elimination and Gauss-Jordan algorithms.
- To develop visual imagery of vectors and operations between vectors in n-dimensional space.
- To develop an understanding and skills of different concepts related to vector space.
- To make students able to understand and apply the concepts of linear maps and isomorphism.
- To develop the idea of representing linear maps with matrices and the relationship between matrices and linear maps.
- To make able to find eigenvalues and eigenvectors of linear maps and matrices.

3. Objectives and contents

Specific Objectives	Contents
• To explain the concept of a matrix and its types of matrices: square matrix, identity matrix, zero matrices, symmetric, skew-symmetric, and triangular matrices with examples. • To explain the concept and properties of addition of matrices, scalar multiplication, and multiplication of matrices and perform these operations.	**Unit I: Matrices and Determinants (4)** 1.1 Introduction of a matrix 1.2 Addition of matrices and its properties 1.3 Scalar multiplication and its properties 1.4 Matrix multiplication and its

• To describe the concept of transpose of a matrix and its properties. • To define the inverse of a matrix and prove some basic theorems. • To describe the determinant of a square matrix and evaluate them • To explain properties of determinants • To find a matrix of adjoint and apply it in finding the inverse of a square matrix.	properties 1.5 Transpose of a matrix and its properties 1.6 Inverse of a square matrix and its properties 1.7 Determinants 1.8 Properties of determinants 1.9 Adjoint of a matrix
• To explain and apply elementary row operations on matrices. • To explain the concept of row equivalent matrices and find the matrices row equivalent to given matrices. • To describe the concept of echelon form matrix and reduced echelon form matrix and find these forms of a given matrix. • To apply the Gauss-Jordan algorithm in finding the reduced row echelon form of a given matrix. • To find the inverse of a square matrix (if exists) using row operations.	**Unit II: Equivalent matrices (4)** 2.1 Elementary row operations on matrices 2.2 Echelon form of a matrix 2.3 Reduced echelon form of a matrix 2.4 Gaus-Jordan algorithm 2.5 Inverse of a matrix using row operations
• To explain the system of linear equations. • To explain the meaning of equivalent linear systems. • To use the Gaussian elimination method to solve linear systems. • To solve linear systems using an augmented matrix of the system. • To determine consistency and inconsistency of linear systems and find solutions if the system is consistent. • To describe the nature of the solution of homogeneous systems and solve them.	**Unit III: System of Linear Equations(4)** 3.1 Introduction of system of linear equations 3.2 Equivalent linear systems 3.3 Gaussian elimination method 3.4 Linear systems and augmented matrices 3.5 Systematic solutions of linear systems 3.6 Homogeneous systems
• To explain the concept of vectors with examples • To find the length of the vector, the sum of vectors, scalar multiplication of vectors, dot product of vectors and angle between vectors. • To state and prove Triangle inequality and Cauchy-Schwarz inequality. • To explain the concept of a linear combination of vectors. • To test whether the given set of vectors is linearly independent or not. • To prove fundamental theorems concerning linear independence of vectors in. • To derive the relationship between linear independence and the system of linear equations and apply the relation in solving problems.	**Unit IV: Vectors in $\mathbb{R}^n$ (8)** 4.1 Introduction of vectors in $\mathbb{R}^n$ 4.2 Length of a vector and angle between vectors in $\mathbb{R}^n$ 4.3 Triangle inequality and Cauchy-Schwarz inequality 4.4 Linear combinations 4.5 Linear independence 4.6 Linear systems and linear independence
• To explain the concept of a field (the field of real numbers). • To define vector space. • To verify that sets of matrices, polynomials, real-valued functions, complex numbers, and Euclidean spaces are vector spaces.	**Unit V: Vector Spaces (10)** 5.1 Concept of a field 5.2 Definition of a vector space 5.3 Subspaces and spanning sets 5.4 Basis and dimensions

• To determine whwther a given set is vector space or not under specified operations. • To define subspace and derive criteria for a subset to be a subspace. • To determine whether a particular subset is a subspace of vector space. • To find spanning sets and determine their linear independence. • To find the basis and dimension of vector space. • Prove fundamental theorems concerning span set, linear independence, basis, and dimensions. • To find row rank, column rank, and rank of a matrix.	5.5 Rank of a matrix
• To define linear maps with examples and test whether particular mappings are linear or not. • To explain the vector space of linear maps. • To define null space, range space, the rank of a liner map, and nullity. • To prove theorems associated with nullity and range space. • To define isomorphism between vector spaces & determine whether a given function is an isomorphism or not. • To prove that vector spaces are isomorphism iff they have the same dimension. • To represent a given linear map with the matrix. • To prove that any matrix represents a linear map. • To find a matrix representing the sum & scalar multiplication of linear maps. • To determine composition & linear maps by using matrix multiplication. • To find a change of basis matrix given bases. • To find map representation concerning a new pair of bases from a given pair of bases. • To find the projection of a vector into a line • To state and prove a theorem of Gram-Schmidt orthogonalization and apply it to find an orthogonal and orthonormal basis for a vector space.	**Unit VI: Linear Maps** (13) 6.1 Concept of a linear map 6.2 Range space and null space 6.3 Nullity and map's rank 6.4 Isomorphism 6.5 Computing Linear Maps 6.6 Representing Linear Maps with Matrices 6.7 Matrix operations and linear maps 6.8 Change of Basis 6.8.1 Changing Representations of Vectors 6.8.2 Changing Map Representations 6.9 Gram-Schmidt Orthogonalization
• To describe the concept of eigenvalue and eigenvectors of matrices. • To find eigen spaces of a matrix corresponding to eigenvalues. • To prove theorems concerning characteristics polynomial. • To find characteristic polynomial of a matrix. • To explain the concept of eigenvalue and eigenvectors of linear maps. • Prove theorems associated with eigenvalue and eigenvectors. • Find eigenvalue and eigenvectors of matrices and linear maps.	**Unit VII: Eigenvalues and Eigenvectors (5 Hrs)** 7.1 Eigenvalue and eigenvectors of a matrix 7.2 The characteristics Polynomial 7.3 Eigen spaces 7.4 Eigenvalue and eigenvectors of linear maps

4. Methodology and Techniques

- Teachers need to focus on concepts, skills, principles, and theorem proving.
- Comparatively more focus should be given in developing skills.
- Follow the suggestions regarding content in the syllabus and recommended books for the uniformity.
- Discuss on the main concepts of the course, engage them in developing skills through examples, and engage them in developing proof of theorems through discussion method.
- Demonstrate the meaning of theorem with help of example and then apply Inquiry Based Learning or any constructivist method to develop proof of the theorems.
- Teachers are suggested to engage students in illustrating theorems by taking suitable examples.
- Assign several questions from the recommended books as homework assignments.
- **Constructivist approach** to develop conceptual understanding of concepts.
- **Project-Based Learning** to facilitate application aspect of theorems in different disciplines.
- Support students in their **ZPD** using constructivist perspective.
- **Exploration**: Help students to explore the essence of the concepts and theorems.
- Emphasize collaborative learning methods.
- Expository-based demonstration methods might be helpful in some content.
- **Discussion:** discuss the application of the theorems/formulas and ask students to solve the problems applying theorems.

5. Evaluation Scheme

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References

Recommended Textbooks

DeFrantz, J. & Gagliardi, D. (2008).*Introduction to linear algebra.* New Delhi, India: Tata McGraw Hill.

Hefferon, J. (2012). *Linear algebra*.

Lang, S. (1986). *Introduction to Linear Algebra*. Springer

Matthews, K. R. (2012). *Elementary linear algebra*.

Reference Books

Chakrabarti, A. (2010). *A first course in linear algebra.* New Delhi, India: Tata McGraw Hill.

Datta, K. B. (2002). *Matrix and linear algebra*. New Delhi, India: Prentice-Hall.

Lipschutz, S.(2000). *Linear algebra.* New Delhi, India:Tata McGraw Hill.

Using Recommended Textbooks:

Unit I [1.1 -1.6 Matthews and Defranza & Gagliardi; 1.7- 1.8 Defranza & Gagliardi; 1.9 Hefferon], Unit II [2.1-2.5 Matthews, 2.5 Defranza & Gagliardi], Unit III [3.1-3.4 Defranza & Gagliardi; 3.5-3.6 Matthews], Unit IV [4.1-4.3 Hefferon; 4.4- 4.6 Defranza & Gagliardi], Unit V [5.1 Matthews; 5.2 Defranza & Gagliardi, Hefferon, and Lang; 5.3 -5.4 Defranza & Gagliardi, 5.6 Hefferon , Lang], Unit VI [6.1 Defranza & Gagliardi, Hefferon; 6.2-6.8 Hefferon], Unit VII [Defranza & Gagliardi , for theorems Lang]