

# Far Western University Faculty of Education Bachelor of Mathematics Education Program

Course Title: **Real Analysis II** Course No: Math.Ed.356 Level: Undergraduate Semester: 5<sup>th</sup> Nature: Theoretical Credits: 3 Teaching Hours: 48

### **1.** Course Description

This course is designed for Undergraduate students to provide fundamental concept of real analysis. Real analysis is fundamental for study of higher mathematics. It attempts to fill the gap and to make transfer from elementary calculus to advance course in analysis. This course deals with limit and continuity, derivability, and Riemann integral of real valued function. The course also includes Riemann Stilljes integral.

#### 2. General Objectives

- 1. To develop the fundamental concept of limits and continuity their role in Real analysis.
- 2. To develop the concept of differentiability and interpret slope of the tangent line in different form.
- 3. To solve optimization problems and apply differentiability in real-world contexts.
- 4. To develop the fundamental concept of Riemann integrable functions and their applications in mathematics.
- 5. To build a strong foundation in real analysis to prepare for advanced mathematical concepts.

Specific Objectives	Contents
• To define function as a relation.	Unit I: Functions(4)
• To explain concept of composite and inverse	1.1 Function as a relation
function.	1.2 Some Specified functions
• To illustrate monotonic real valued functions with	1.3 Composite functions and inverse
<ul><li>example.</li><li>To illustrate Monotonic real valued function at a</li></ul>	functions
point	1.4 Functions with range in $\mathbb{R}$
• To explain $\mathcal{E} - \delta$ definition of a limit of a	Unit II: Limit of a functions (6)
function.	2.1Definition of a limit of a function
• To explain limit of a function graphically.	2.2 Algebra of limits of a function
• To prove Algebraic properties of limit of a	2.3 One sided limits
<ul><li>function.</li><li>To illustrate concepts of one sided limits and their</li></ul>	2.4 Infinite limits and limits at infinity
graphical representation.	2.5 Upper and lower limits of
• To explain technique of evaluation of one sided	indetermination
limits.	
• To discuss with examples concept of infinite limits	
and limits at infinity.	
• To discuss with examples concept of Upper and	
lower limits of indetermination	

#### **3.** Specific objectives and contents

<ul> <li>To explain concept of continuity of a function at a point on its domain.</li> <li>To illustrate concepts of composition of Continuous function</li> <li>To classify discontinuities of functions at a point.</li> <li>To prove some important theorems on continuity.</li> <li>To discuss with examples concept of Uniform continuity of functions graphically and prove their theorems</li> </ul>	<ul> <li>Unit III: Continuity of a functions (11)</li> <li>3.1 Definition of continuity of a function at a point</li> <li>3.2 Continuity of a function of a function</li> <li>3.3 Discontinuity of a function and its types</li> <li>3.4 Theorems on continuity of functions(Borel's Theorem, Boundedness Theorem, Intermediate value theorem and Fixed point theorem)</li> <li>3.5 Uniform continuity of functions</li> </ul>
<ul> <li>To explain concept of derivative of a function at a point.</li> <li>One sided derivability of a function at a point</li> <li>To explain relation between continuity and derivability.</li> <li>To prove Algebraic properties of derivative.</li> <li>To state and prove Mean value theorems and interpret them graphically.</li> <li>To state and prove Taylor's and Maclaurin's theorem.</li> <li>To discuss different indeterminate forms with examples.</li> <li>To prove theorems on indeterminate forms including L' Hospital's rule.</li> </ul>	Unit IV: Derivability (13) 4.1 Definition of a derivative of a function 4.2 Derivability and continuity 4.3 Algebraic Properties of derivatives 4.4 Mean value theorems (Rolle's theorem, Lagrange's Mean value theorem, Cauchy's mean value theorem) 4.5 Taylor's theorem and Maclaurin's theorem 4.6 Indeterminate forms $(\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 0^0, \infty^0, 1^\infty)$ and L' Hospital's rule.
<ul> <li>To define Upper and Lower Darboux sums and prove four properties.</li> <li>To define upper and lower Riemann integral of a function on closed interval and prove Darboux theorems</li> <li>To explain concept of Riemann integral of a function.</li> <li>To state and prove Necessary and sufficient condition for integrability.</li> <li>To prove properties of Riemann integral and integrable functions.</li> <li>To state and prove generalized mean value theorems</li> <li>To state and second fundamental theorem of integral calculus.</li> <li>To explain the technique of evaluating the definite integral using integration by parts.</li> </ul>	<ul> <li>Unit V: Riemann Integration (14)</li> <li>5.1 Darboux sums and Four properties</li> <li>5.2 Upper and Lower Riemann integral</li> <li>5.3 Definition of Riemann integral</li> <li>5.4 Necessary and sufficient condition for integrability.</li> <li>5.5 Properties of Riemann integral</li> <li>5.6 Properties of integrable functions</li> <li>5.7 Generalized Mean value theorems for integral</li> <li>5.8 Continuity and Derivability of Function defined by means of Integral</li> <li>5.9 Integration by parts</li> </ul>

### 4. Methodology and Techniques

Teaching methods include both teacher-centered and student-centered approaches, carefully chosen based on the nature of the content. These methods emphasize delivering detailed explanations supported by examples and visual aids to enhance concept clarity. Additionally, they promote group discussions, teamwork, problem-solving approaches, presentations, and collaborative learning techniques.

#### 5. Evaluation Scheme

The assessment of student's performance is made through formative and summative evaluation. Classroom activities, report writing, presentation, individual work and group work can be used as formative evaluation. For summative evaluation internal assessment of 40% and external evaluation of 60% will be conducted. Internal assessment should be used as formative evaluation also.

#### 5. Evaluation Scheme

#### 5.1 Internal Evaluation 40%

Internal Evaluation will be conducted by course teacher based on following activities.

a) Attendance and Participation in class activities:	5+5=10
b) Assignment I: Reflective Notes and Class presentation:	5+5=10
(Reflective notes on 2 to 4 questions given by teacher at the end	
of the every unit and presentation on any two questions among them )	
c) Assignment II: an Term paper/ Essay/Project and Interview:	5+5=10
(Logical essay/term paper/project on the topics chosen by students	
and approved by the teacher and interview)	
d) Mid-term exam:	10

#### 5.2 External Evaluation (Final Examination) 60%

Office of the Controller of Examination will conduct final examination at the end of semester.

Types of questions	Total questions to be asked	Number of questions to be answered and marks allocated	Total marks
Group A: Multiple choice items	10 questions	10 × 1	10
Group B: Short answer questions	6 with 2 'or' questions	6×5	30
Group C: Long answer questions	2 with 1 'or' question	2×10	20

#### References

Gupta, S. L. and Rani, N. (2003).*Fundamental real analysis* (4<sup>th</sup>). New Delhi: Bikash Publishing House Maskey, S.M. (2007). *Principles of real analysis*. (2<sup>nd</sup>). Kathmandu: RatnaPustakBhanda

Malik, S.C. and Arora, S. (2010). *Mathematical analysis* (4<sup>th</sup>). New Delhi: New Age International Pvt. Ltd



Far Western University Faculty of Education Bachelor of Education Program

Course Title**: Topology and Graph Theory** Course code: Math.Ed.357 Level: Undergraduate Semester: 5<sup>th</sup> Nature: Theoretical Credits: 3 Teaching Hours: 48

### 1. Introduction:

Graph Theory is one of the branches of Modern Mathematics. It deals in solving net-working problems of modern scientific world. It is frequently applied in physics Mathematics, Engineering, Biology, chemistry, geography, and many other subjects. An engineer uses a planar graph theory out the plan of utility services (such as supply of water, electricity, gas etc) to different houses of urban area. A tourist wishing to visit famous cities of the world may use the shortest path problems to make his tour the most economical. Euler's theorem of graph theory is used in surface of polydedra.

### 2. General objectives:

- a. To make the students familiar with the net-working problems of the scientific world.
- b. To make them `able to apply the knowledge of graph theory of different scientific subjects.
- c. To make them able to use graph theory to solve the net-working problems that arises in their daily life.
- d. To apply the graph theory on surface of convex polyhedral.
- e. To identify the topologically equivalent surfaces such as one-sided surface and two- sided surfaces.
- 3. Specific objectives and Contents

# Unit 1: Introduction and basic concepts of graph. (10)

examples.

•	To represent given graph in matrix forr	
	(incidence, adjacency) and vice-versa.	

# Unit 2: Traversability (9)

<ul> <li>Contents</li> <li>2.1.Eulerian and Hamiltonian graphs.</li> <li>2.2.Konigsberg Bridge problem, Eulerian circuit with even vertices, Eulerian circuit with two odd vertices.</li> <li>2.3.Theorems on existence of Eulerian graphs. (only two theorems)</li> <li>2.4.Hamiltonian path, Hamiltonian cycle.</li> <li>2.5.Weighted graph, shortest path problem, chinese postman problem</li> </ul>
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$\mathbf{U}_{\mathbf{r}}$
Unit 3: Trees (6)
Contents
<ul><li>3.1. Definition and examples of tree</li><li>3.2. Properties of trees</li><li>3.3. Spanning trees</li><li>3.4. Minimal spanning trees.</li><li>3.5. Kruskal's algorithm</li></ul>
Unit 4: Planar Graphs (10)
Contents
<ul> <li>4.1. Planar graph definition and examples.</li> <li>4.2. Regions of planar graphs.</li> <li>4.3. Euler's theorem v –e +r =2</li> <li>4.4. Theorem on Relation of number of edges and regions of planar graphs.</li> <li>4.5. Theorem on number of edges and vertices</li> <li>4.6. Colouring of planar graph</li> <li>4.7. Colouring of maps</li> <li>4.8. Chromatic number</li> <li>4.9. Weich-Powell algorithm for colouring of graphs.</li> <li>4.10. Five colourable theorems.</li> </ul>

# Unit 5: Topology of surfaces and Euler's theorem (13)

Specific objectives	Contents

	5.1. Topological equivalence
• To define topologically equivalence the	5.2. Polyhedra
surfaces	5.2.1. Convex polyhedron
	5.2.2. Regular polyhedron
• To define convex polyhedron	5.2.3. Euler's theorem on convex
• To prove Euler's theorem on	polyhedron
polyhedron.	5.2.4. Theorem on number of regular
• To prove the theorem on number of	polyhedra
polyhedral.	5.3. Dual of a polyhedron
<ul> <li>To identify two sided and one-sided</li> </ul>	5.4. Semi regular polyhedron
-	5.5. Topology of surfaces
surfaces.	5.5.1. Two sided surfaces without boundary
• To define dual of a polyhedron.	5.5.2. Torus
• To define Torus and two-fold Torus.	5.5.3. Two-fold Torus
• To construct model of torus and Klein	5.5.4. Genus of a surface
bottle.	5.6. Euler's characteristics of a surface.
bottic.	5.7. Theorem on V-E+F = $2-2p$
	5.8. One sided surface

### 1 Methodology and Techniques

- a. Teachers are suggested to facilitate students by visualizing every concept of graph.
- b. Teachers are suggested to clarify the application part in each topic.
- c. Engage students in drawing activities and establishing link between diagrams and definitions or verbal explanations.
- d. Promotes students for inquiry-based learning to develop proof of the theorems.
- e. Constructivist approach to develop conceptual understanding of concepts.
- f. Problem Based Learning to help students in solving problems in the exercises.
- g. Exploration: Help students to explore the essence of the concepts and theorems.
- h. Use collaborative learning methods together with expository-based demonstration methods as per the nature of the content.
- i. Discussion: discuss the application of the theorems/formulas and ask students to solve the problems applying theorems.
- j. Teachers may use mathematical software Geogebra.

#### 2 Evaluation Scheme

2.2 **Internal Evaluation** (40%): Internal Evaluation will be conducted by course teacher based on following activities.

a.	Attendance and Participation in class activities:	5+5=10 marks
b.	Assignment I: Reflective Notes and Class presentation:	5+5= 10 marks
	(Reflective notes on 2 to 4 questions given by teacher at the end of every unit and presentation on any two questions among them)	
c.	Assignment II: one Term paper/ Essay/Project and Interview:	5+5=10 marks
	(Logical essay/term paper/project on the topics chosen by	
	students and approved by the teacher and interview)	

10 marks

d. Mid-term exam:

#### 2.3 External Evaluation (60%)

Nature of questions	Total questions to be asked	Total marks
Group A: Multiple choice	10	10x1=10
Group B: Short questions	6 questions with 2 or	6x5=30
	questions	
Group C: Long questions	2 questions with one or	2x10=20
	question	
Total		60

### **3** Prescribed Books and References

Chartrand, G & Zhang, P. (2012). First course in graph theory. Dover Publications, Inc. (unit 1-5) <u>http://lib.ysu.am/disciplines\_bk/</u> or <u>https://store.doverpublications.com</u>

Maskey, S.M. (1995). Introduction to modern mathematics. Volume 2. Ratan Pustaka prakasan.

Maskey, S.M. (1998). First course in graph theory. Ratan Pustaka prakasan.



# Far Western University Faculty of Education Bachelor of Mathematics Education Program

Course Title: Designing Instructional Materials for Teaching Mathematics

Course No. : Math.Ed.358 Level: Undergraduate Semester: 5<sup>th</sup> Nature of course: Theory Credit Hour: 3 Total periods: 48 Hrs

## **1.** Course Introduction

This course is designed for undergraduate students to develop skills in developing and using instructional materials in teaching school-level mathematics. The course starts with the introduction of teaching materials, progresses through content-wise teaching materials, and ends with digital resources. The course focuses on developing a deeper understanding of several mathematical concepts, deriving some formulae, and verifying some mathematical principles through teaching materials. The course is designed in such a way that it helps teachers to facilitate mathematical content in a meaningful way. Meaningful learning and conceptual understanding of school-level mathematics are taken into consideration. The major focus of the course is on practical work and skill development.

# 2. Course Objectives

At the end of the course, the students are expected to achieve the following objectives:

- To realize the importance of mathematics laboratory and instructional materials in mathematics teaching.
- To demonstrate skills in designing models of teaching concepts, relations, and formulae of different domains of mathematics.
- To demonstrate understanding and skills in using instructional materials in mathematics teaching.
- To show understanding and skills in applying digital resources in mathematics teaching.
- To give value to meaningful learning, conceptual understanding, and practical knowledge of mathematical contents.
- To appreciate the value of instructional resources in mathematics teaching.

# 3. Course Contents and Specific Objectives

Specific Objectives	Content
• To explain the concept, types, and	Unit I: Instructional Materials and
rationale of instructional materials used in	Mathematics Laboratory [3]
mathematics teaching.	1.1 Introduction of Instructional materials
• To explain the concept and purpose of	1.2 Types of instructional materials
mathematics laboratory.	1.3 Concept of Mathematics Laboratory
	1.4 Purpose of mathematics laboratory
• To explain the construction process of	Unit II: Materials for Teaching Mensuration[13]
models (made by card sheet, plywood,	2.1 Models to teach area of plane figures
glaze paper, transparent sheet, cardboard,	

<ul> <li>thermocol sheet, or similar objects) for deriving/demonstrating formula for area of plane figures.</li> <li>To design above mentioned models and explain their application in mathematics teaching.</li> <li>To demonstrate formula for area of plane figures by paper folding and cutting.</li> <li>To explain the meaning of nets, solid structure, and skeleton model of 3D figures.</li> <li>To construct nets, solid structure, and skeleton model of specified solids.</li> <li>To derive and demonstrate the formula for surface area and volume of 3D figures using models.</li> <li>To develop models representing volume relationship between specified solids.</li> </ul>	<ul> <li>2.1.1 Models to derive/demonstrate formula for area of plane figures [triangle, parallelogram, rhombus (in terms of diagonal), square (in terms of diagonal), trapezium, kite, quadrilateral, aero head, circle]</li> <li>2.1.2 Demonstration of formula for area of plane figures (mentioned in 2.1.1) by paper folding and cutting.</li> <li>2.2 Three dimensional objects, their surface area and volume</li> <li>2.2.1 Meaning of Nets, solid structure, and skeleton of 3D objects.</li> <li>2.2.2 Nets and solid structure of five regular polyhedron, prism (triangular, rectangular, pentagonal, L shaped, stepped, plus shaped, E shaped), pyramid (triangular, square based, pentagonal), circular cylinder, and circular cone</li> <li>2.2.2 Skeleton model of five regular polyhedron, cuboid, prism and pyramid</li> <li>2.2.3 Models to derive/demonstrate surface area and volume of cylinder, cone, frustum of a cone, prisms, pyramids, sphere, hemisphere, hollow sphere, hollow cylinder</li> <li>2.2.4 Models to represent volume relationship between cone and cylinder; between right circular cylinder of same height and radii; between prism</li> </ul>
	and pyramid
<ul> <li>To describe the construction process of Mecano strips, Tangram, Geoboard, Circle board, coordinate wheel, and coordinate board and their application in mathematics teaching.</li> <li>To develop models used to verify various specified results (mentioned in content column) and demonstrate the application models in teaching those results.</li> <li>To identify incentre, circumcenter, centroid, orthocenter and demonstrate their relation by paper folding.</li> </ul>	<ul> <li>Unit III: Materials for Teaching Geometry [8]</li> <li>3.1 Mecano strips, Tangram, Geoboard (square and circular), Circle board, coordinate wheel, coordinate board,</li> <li>3.2 Models used to verify various theorems</li> <li>[concerning parallel lines intersected by a transversal; sum of the angles of a triangle; sum of the angles of a triangle of a triangle, sum of exterior angles of polygon; congruence theorems (SSS, SAS, ASA, RHS); opposite angles of cyclic quadrilateral; Pythagoras theorem]</li> <li>3.3 Paper folding and cutting: Incentre, circumcenter, centroid, orthocenter and their relation</li> </ul>
• To develop base 10 blocks, Cuisanaires Rod/Number strips,	Unit IV: Materials for Teaching Arithmetic [5] 4.1 Base-10 Blocks

1.2 Cuiconairea Dad/Number string
4.2 Cuisanaires Rod/Number strips
4.3 Number lines, number plates and Balance Beam
4.4 Material for teaching base 2 numeration system
4.5 Magic squares of type 3*3 and 4*4
4.6 Models for finding HCF and LCM
4.7 Fractional Kit
Unit V: Materials for Teaching Algebra [6]
5.1 Introduction of algebraic tiles
5.2 Addition and subtraction of like terms using tiles
5.4 Factorizing using algebraic tiles
5.5 Verification of algebraic identities geometrically
$[(a+b)^2 = a^2 + 2ab + b^2, (a-b)^2 = a^2 - 2ab + b^2, (a+b)^3$
$=a^{3}+3a^{2}b+3ab^{2}+b^{3}, (a-b)^{3}=a^{3}-3a^{2}b+3ab^{2}-b^{3},$
$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca, a^3-b^3=(a-b)$
$(a^2+ab+b^2)$ , $a^3+b^3 = (a+b)(a^2-ab+b^2)$ ]
Unit VI: Materials for Teaching Trigonometry[3]
6.1 Clinometer (30°, 45°, 60°, and general)
6.2 Hypsometer
6.3 Trundle wheel
6.4 Trigonometric ratio explorer using circle board
6.5 Scientific model of sine, cosine, and tangent
ratio.
Unit VII: Digital Resources for Teaching
Mathematics [10]
7.1 LaTeX
7.2 GeoGebra
7.3Mathigon
7.4Mathematica
7.5 Maple
7.6MatLab
7.7 Desmos

## 4. Methodology and Techniques

• To facilitate model/materials construction, start the class with a discussion on the theoretical part, required raw materials, and the construction process of the material.

- After sufficient discussion let the students work in groups or individually according to the nature of the model.
- Support students during the material construction.
- After the construction of materials let them explore the use of the materials in mathematics teaching.
- Ask students to demonstrate the construction process and application of materials they developed.
- Let students keep a record of all the practical works (including the name of the material, required raw materials, construction process, and use in math teaching) completed during the class discussion, individual homework, and group homework assignments.
- As far as possible, manage one room for the math laboratory, and keep all the materials in that lab after designing the materials.
- As far as possible encourage group work and ensure collaboration.
- Project-based learning methods might work on several topics.
- To facilitate unit VII, discuss the introduction and application of specified digital resources. Discuss the way of searching readymade materials in different software. Give more emphasis on the Geogebra learning resources. Engage students in developing digital resources using GeoGebra.

## 5. Evaluation Scheme

### 5.1 Internal Evaluation (40%)

Internal Evaluation will be conducted by course teacher based on following activities.

a) Attendance and Participation in class activities	4+4= 8 marks
b) Assignment I: Designing Material and its presentation	3+3= 6 marks
(Student will design assigned teaching material and demonstrate	
<ul><li>construction process and application)</li><li>c) Assignment II: Group Project work</li></ul>	4+4=8 marks
(Students will be provided a project work group wise; they have to	4+4–0 mai K5
prepare project report, and present in the classroom)	
d) Mid-term exam:	8 marks
5.2 External Evaluation (600/)	

### **5.2 External Evaluation (60%)**

External evaluation will consist of two categories: written examination covering 40 marks and practical covering 20 marks.

(1) Written Examination: It is also a written examination, and the questions will be asked covering all the topics in the session of the course. It carries 40 marks.

Nature of question	Total questions to be asked	Total questions to be answered	Total marks
<b>Group A</b> : Multiple choice	8 questions	10	10×1 = 10
<b>Group B:</b> Short answer type question	6 with 2 'or' questions	6	6×5= 30

### **End Semester Examination Model**

<b>Group C:</b> Long answer type question/case studies	1 with 'or' question	1	1×10 =10
	Total		50

(2) **Practical** (20%): Office of the controller of examination will conduct final practical examination after the end of the final written examination. External examiner will conduct practical examination and the internal examiner will assist the external examiner. The evaluation will be based on the following criteria.

Category	Internal	External	Total
Copy of all the works done during semester with sample	3	2	5
materials (3-5 materials)			
Viva based on the in-semester works	2	3	5
Practical Examination in Math Lab of concerned campus	3	7	10
(designing any one materials asked by external examiner)			
Total	8	12	20

#### **Recommended Textbooks**

Gratzer, G. (2024). Text and math into latex. Springer [for unit VII]

- Hall, J., & Lingefjärd, T. (2016). *Mathematical Modeling: applications with Geogebra*. John Wiley & Sons. [for unit VII]
- Khanal, B., Roka, J., Khatri, T. (2020). *Teaching material development*. Cambrize Publication [Optional; Teacher may use for some topics]
- Maharjan, H. B., Paudel, L., & Upadhyay, H. N. (2068). *Teaching mathematics in secondary schools*. Buddha Publications, [for unit I]
- Maharjan, H. B., & Upadhyay, H. N. (2009). *Mathematics instructional materials*. Paluwa Prakashan [for units I-VI]
- Singh, H., Avtar, R., & Singh, V. P. (2011). *A handbook for designing mathematics laboratory in schools*. National Council of Education Research and Training. [for units I-VI]
- Torrence, B. F., & Torrence, E. A. (2019). *The student's introduction to mathematica and the wolfram language*. Cambridge University Press. [for unit VII]
- Upadhyay, H. N. (2002). *Folding and cutting of papers in the teaching of mathematics*. Ekta Books Distributor Pvt. Ltd. [for the content related to paper folding and cutting]
- Upadhyay, H. P., Uadhyay, M. P., & Luitel, S. (2070).*Exploratory teaching mathematics*. Sukunda Pustak Bhawan [for unit I]